Analysis of impact of the subprime crisis in the Brazilian agribusiness firms listed on BM&FBovespa.

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Abstract

This study examines the effects of exhaustible resource inputs. By establishing a dynamic game, this paper argues that the price of the end product increases over time, while the exhaustible resource input decreases over time. Firstly, firms with lower efficiencies quit this industry early, enabling researchers to calculate a firm’s “quitting time.” Secondly, the exhaustible resource input and profits of all firms decrease with the number of firms. Finally, in an asymmetric case, the degree of production efficiency between firms with higher efficiency and lower efficiency has major effects on the firms’ strategies.

Keywords: Exhaustible resource. Oligopoly. Quitting time.

1. Introduction

Many types of production require inputs of exhaustible resources. This exhaustible resource affects firms’ products and prices. What effect does demand have on this exhaustible resource? How does one determine the price of the outputs? These questions motivate this study of oligopolies and exhaustible resources.

Gejiu City of Yunnan Province in China, is famous because of its rich tin reserves (http://en.wikipedia.org/wiki/Gejiu and http://en.wikipedia.org/wiki/Tin). In Gejiu City, the industry to explore tin heavily depends on the tin mineral stock, and the tin mineral is a type of exhaustible resource. In recent years, along with the reduction of the tin mineral stock, many firms quit this industry sequentially (http://www.chinacity.org.cn/cspp/cspp/60988.html). This example illustrates that the exhaustible resource has crucial impact on the industry depending on the exhaustible resource. This motivates our further research on this topic. This study addresses the effects of exhaustible resource on the production price and outputs both under the symmetric situation.
Exhaustible resources attract extensive attention in many fields. Bahel (2011) developed an optimal management strategy for strategic reserves of nonrenewable natural resources. Bems and de Carvalho (2011) addressed the current account and precautionary savings of exhaustible resources. Gerlagh and Liski (2011) discussed the strategies regarding exhaustible resource management. Liski and Montero (2011) discussed the impact of market power on exhaustible resource inputs, including the use of pollution permits. Van der Ploeg (2010) investigated the competition over exhaustible resources and argued that property right of exhaustible resources has a major effect on firms’ strategies.

Some empirical evidence indicates that exhaustible resources have major effects on the economy. Boyce and Emery (2011) confirmed that the correlation between growth and natural resource abundance is negative. In another empirical paper on exhaustible resources, Boyce and Nostbakken (2011) examined the exploration and development of oil and gas fields in the United States over the period 1955-2002 and explained the economic evolution of the exhaustible resource industry.

Acemoglu et al. (2012) established a dynamic model to address technological innovation with exhaustible resource inputs. This paper introduced a production function with human capital and an exhaustible resource. Because an exhaustible resource is continuously reduced in production, it is rational to model it with a dynamic model, the same method used by Liski and Montero (2011).

The subject of exhaustible resource inputs is a challenging topic because it covers the environment, resources and regulation. Moreover, exhaustible resources have major effects on firms’ strategies (Gerlagh & Liski 2011). This study focuses on oligopolies of exhaustible resource and establishes a model with reference to the interesting dynamic model in Acemoglu et al. (2012). Due to the property of exhaustible resources, this study stresses the quitting time of firms, which sets it apart from Acemoglu et al. (2012). Comparing the quantity of exhaustible resource inputs at each stage yields some interesting conclusions. This study limits its scope to a single type of product, while Acemoglu et al. (2012) discussed a “clean” product and a “dirty” product. The model of this paper refers to the interesting paper of Van der Ploeg (2010).

This paper is organized as follows. The model, which fully considers a certain type of exhaustible resource, is established in the next section. Section 3 addresses this model and the
concept of quitting time. Section 4 analyzes the model in the context of the number of firms in that industry. Some concluding remarks are presented in the final section.

2. Model

There are \( N \) firms in an industry that produce identical products. The number of firms is denoted by \( N = \{1, 2, \ldots, N\} \). In this industry, to produce the corresponding products requires a certain exhaustible resource. This industry produces a unique output. Given the price \( p_t \), demand is given by \( D_t = A - p_t \), where \( A \) is a constant. A linear demand function is employed to simplify this model. The production function of firm \( i \) at time \( t \) is given as follows.

\[
q_i^t = \theta_i (er_i^t)^\alpha,
\]

where \( \theta_i > 0 \) stands for the production efficiency of firm \( i \). Without the loss of generality, we assume that \( \theta_1 \geq \theta_2 \geq \cdots \geq \theta_N \). The notation \( er_i^t \geq 0 \) describes the exhaustible resource input at time \( t \) for the firm \( i \), and \( 0 < \alpha \leq 1 \) is a constant. There are other inputs involved in production, but these inputs are ignored in this study for the purpose of focusing on the exhaustible resource input. Therefore, the production function entirely depends on the exhaustible resource.

The stock of the exhaustible resource at time \( t \) is \( S_t \). At the initial stage, the stock of this exhaustible resource is \( S_0 \). Obviously, \( S_t = S_0 - \int_0^t \left( \sum_{i=1}^N er_i^t \right) dt \) or \( \frac{dS_t}{dt} = -\sum_{i=1}^N er_i^t \) for \( 0 \leq t \). This formulation of the stock of exhaustible resources at each stage also appears in Liski and Montero (2011) and Acemoglu et al. (2012).

The marginal costs of the exhaustible resource at time \( t \) are \( c(S_t) \). The marginal costs \( c(S_t) \) increase as the stock of the exhaustible resource is depleted. Moreover, \( c' < 0 \) and \( c'' > 0 \). \( c(S_t) \) is exogenously determined by multiple factors. This assumption of marginal costs is consistent with the empirical results in Boyce and Nostbakken (2011) and Acemoglu et al. (2012).
Market clearing conditions imply the relationship $D_t = \sum_{i=1}^{N} q_i$. The game stops at time $T$ if all firms quit this industry. We note that $T$ is addressed in Section 3. For $i = 1, 2, \ldots, N$, the total profits of firm $i$ are

$$\pi_i = \int_{0}^{T} \left[ \int_{0}^{T} \left[ (A - \sum_{i=1}^{N} \theta_i(q_i) d\theta_i) - c_i(q_i) \right] dt \right] .$$

The discounting factor for all firms is assumed to be 1 to simplify the model. The unit cost of the exhaustible resource varies with the stock of exhaustible resource; namely, it decreases as the stock of exhaustible resource decreases.

The sequence of the game is outlined as follows. At the initial stage, all firms know the stock of the exhaustible resource, its marginal price, the production efficiency of all firms and the demand in this industry. All firms determine their exhaustible resource inputs for all $t$. When $p_t q_i - c(S_t) \leq 0$ for firm $i$ at time $t (i = 1, 2, \ldots, N)$, firm $i$ quits this industry. Finally, all firms quit this industry at time $T$, thereby ending the game.

This paper considers finite horizon, which is different from that of Van der Ploeg (2010). Moreover, the discount factor is assumed to be 1 to simplify the model. Firstly, in this work, firms quit this industry if the resource is very scarce.

At the last stage, for the steady state, all firms quitting this industry. Moreover, this paper considers the production costs while Van der Ploeg (2010) did not care about the production costs. Therefore, it is rational to employ the finite horizon. Secondly, compared with the interesting paper of Van der Ploeg (2010), this paper considers the common asset. The property right of exhaustible resources is not focused by this work. Finally, in the finite horizon, under the common asset, it is rational to assume the discount factor to be 1 because the discount factor has no effects on the equilibrium.

3. Results

This section analyzes the above model, restated as follows:
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\[ \text{Max}_{q_i} \pi_i' = \int_0^T [p_i q_i' - c(S_i) er_i'] \, dt = \int_0^T \left[ A - \sum_{i=1}^N \theta_i (er_i')^\alpha \right] \theta_i (er_i')^\alpha - c(S_i) er_i' \, dt \]

\[ \text{S.t.} \quad \frac{dS_i}{dt} = -\sum_{i=1}^N er_i' . \]

Denote the profit of firm \( i \) at stage \( t \) to be \( \pi_i' = [A - \sum_{i=1}^N \theta_i (er_i')^\alpha \theta_i (er_i')^\alpha - c(S_i) er_i' \]. With model (3) now characterized, the existence of a unique solution is obtained below.

**Proposition 1:** There exists a unique solution to (3).

**Proof:** This conclusion is immediately achieved by directly calculating the second order derivative under \( 0 < \alpha \leq 1 \). □

This section addresses the model further by first characterizing the equilibrium and then discussing quitting time.

### 3.1. Equilibrium

Model (3) is a type of fixed endpoint problems, which is also a type of Euler equation (Kamien & Schwartz 1991, p. 147). If the solution is a strictly interior point, the optimal conditions of (3) are outlined as follows. The proof in detail is deleted, which refers to the interesting monograph (Kamien & Schwartz 1991).

\[ \frac{\partial}{\partial er_i'} \{A - \sum_{i=1}^N \theta_i (er_i')^\alpha \theta_i (er_i')^\alpha - c(S_i) er_i' \} - \lambda_i' \]

\[ = \alpha \theta_i (er_i')^\alpha \left[A - \sum_{i=1}^N \theta_i (er_i')^\alpha - \theta_i (er_i')^\alpha \right] - c(S_i) - \lambda_i' = 0 . \]

The Lagrangian multiplier \( \lambda_i' \geq 0 \) for all \( t \) is given by the following equation:

\[ \frac{d \lambda_i'}{dt} = -\frac{\partial}{\partial S_i} \left[ A - \sum_{i=1}^N \theta_i (er_i')^\alpha \theta_i (er_i')^\alpha - c(S_i) er_i' \right] = er_i' \frac{\partial c(S_i)}{\partial S_i} . \]
In economics, for (4), \( \lambda_i^t \geq 0 \) means the marginal profits of firm \( i \) at time \( t \). Apparently, (5) indicates that the marginal profits decrease with time. Further denote \( f_i = \alpha \theta_i (er_i)^{\alpha - 1} \left[ A - \sum_{j=1}^{N} \theta_j (er_j)^{\alpha} - \theta_i (er_i)^{\alpha} \right] - c(S_i) - \lambda_i^t = 0 \). Obviously, \( \frac{\partial f_i}{\partial er_i} < 0 \) and \( \frac{\partial f_i}{\partial \theta_i} > 0 \) if \( c(S_i) \) is large enough. Because it is very difficult to achieve the explicit solution, we analyze dynamic results with implicit function theorem or total differential method. From (4) and (5), by the implicit function theorem or total differential method to (4), for \( i = 1, 2, \ldots, N \), we have

\[
\frac{\partial er_i}{\partial \theta_i} = -\frac{\frac{\partial f_i}{\partial \theta_i}}{\frac{\partial f_i}{\partial er_i}} > 0 \quad \text{and} \quad \frac{\partial er_i}{\partial \alpha} = -\frac{\frac{\partial f_i}{\partial \alpha}}{\frac{\partial f_i}{\partial er_i}} > 0
\]

for a sufficiently small stock of the exhaustible resource. Higher efficiency firms are more aggressive in competition, and firms with lower efficiency are more conservative in competition. This means that higher efficiency firms require more exhaustible resource inputs. This is rational and indicates that the firms with higher efficiency have larger outputs.

When the stock of the exhaustible resource is large enough, \( c(S_i) \) and \( \lambda_i^t \) are all small enough. The conclusion may be different. Obviously, \( \frac{\partial f_i}{\partial er_i} < 0 \) and \( \frac{\partial f_i}{\partial \theta_i} < 0 \) if \( c(S_i) \) is small enough. Therefore, for \( i = 1, 2, \ldots, N \), we have

\[
\frac{\partial er_i}{\partial \theta_i} = -\frac{\frac{\partial f_i}{\partial \theta_i}}{\frac{\partial f_i}{\partial er_i}} < 0 \quad \text{and} \quad \frac{\partial er_i}{\partial \alpha} = -\frac{\frac{\partial f_i}{\partial \alpha}}{\frac{\partial f_i}{\partial er_i}} > 0
\]

and \( \lambda_i^t \) are all very small. Firms are more competitive under that large stock of the exhaustible resource than that when the exhaustible resource is scarce. In this case, the exhaustible resource has no major effect on the production of firms.

Moreover, by a similar method, we immediately have \( \frac{\partial er_i}{\partial \theta_j} < 0 \) for \( i, j = 1, 2, \ldots, N \) and \( i \neq j \). This means that the higher production efficiency of one firm reduces its rival’s exhaustible resource input and corresponding output of products.

Obviously, (4) implies \( \lim_{S_i \to 0} c(S_i) = +\infty \) and \( \lim_{S_i \to 0} er_i = 0 \). All firms finally quit this industry because of increasing marginal cost. According to Proposition 1, the equilibrium is entirely determined by (4) and (5). From (4) and (5), we have:
Proposition 2: Price $p_t$ monotonically increases in $t$, while exhaustible resource input $er^i_t$ monotonically decreases in $t$ for all $i = 1, 2, \ldots, N$. Correspondingly, $\pi^i_t$ monotonically decreases in $t$ for all $i = 1, 2, \ldots, N$.

Proof: See Appendix. ■

Remarks: This conclusion implies that the price of these products continuously increases, while the exhaustible resource input continuously decreases. This result is due to the nature of exhaustible resources and differs from a general situation without exhaustible resource constraints. This conclusion is consistent with some social phenomena. In China, for example, gasoline, which is a type of exhaustible resource, has been subject to continuous price hikes in recent years.

The profit function under equilibrium must be addressed. By the envelop theorem, combining the above conclusions $\frac{\partial er^i_t}{\partial \theta^i} > 0$ and $\frac{\partial er^i_t}{\partial \alpha} < 0$, we immediately have the relationship $\frac{\partial \pi^i_t}{\partial \theta^i} > 0$ and $\frac{\partial \pi^i_t}{\partial \alpha} > 0$. Taking exhaustible resources into account, firms benefit from higher production efficiency. This conclusion is consistent with cases even without exhaustible resource inputs.

3.2. Quitting

Because of the exhaustible resource, the marginal costs continuously rise with time. This makes the quitting time of firms an important concept because the various production efficiencies of firms yield different quitting times. By proposition 2, the price of the products increases, while the exhaustible resource input and profits ($\pi^i_t$) decrease with time. According to the proof of proposition 2, for time $t$ and firm $i$ we have the relationship $\frac{\partial \pi^i_t}{\partial \theta^i} > 0$. Because its profits at time $t$ are less than zero, when $p_t q^i_t - c(S_t) er^i_t \leq 0$, firm $i$ quits this industry. In this case, firms quitting this industry will not reenter it, according to Proposition 2. This case is also equivalent to solving the maximization problem for firm $i$ at time $t$ lying at the corner.
\[ \frac{\partial \pi_i}{\partial \theta_i} > 0 \] and Proposition 2 indicate the following conclusion: Firm \( N \) first quits this industry. Then, firm \( N-1 \) quits. Eventually, the last firm quits this industry. When the first firm quits this industry, it should satisfy the relationship \[ \alpha \theta_i (er_i^t)^\alpha [A - 2 \theta_i (er_i^t)^\alpha] - c(S_r) = 0 \] and \[ A \theta_i (er_i^t)^\alpha - \theta_i^2 (er_i^t)^{2\alpha} - c(S_r)er_i^t \leq 0 \] while \[ \alpha \theta_i (er_i^t)^\alpha [A - 2 \theta_i (er_i^t)^\alpha] - c(S_r) = 0 \] and \[ A \theta_i (er_i^t)^\alpha - \theta_i^2 (er_i^t)^{2\alpha} - c(S_r)er_i^t \geq 0 \] for all \( t < T \) hold.

**Remarks:** According to the above conclusion, firms quit this industry in order according to their production efficiency. Firms with lower efficiency quit earlier. At the last stage, all firms quit this industry. As some firms quit this industry, competitive pressure is reduced, while production costs rise.

When firm \( i \) determines to quit this industry, we have \( p_i q_i^t - c(S_r)er_i^t = 0 \). By the continuity of the corresponding profit function, there exists a unique solution to this equation. This is rewritten as follows:

\[ [A - \sum_{j=1}^{L} \theta_j(er_j^t)^\alpha]q_i^t - c(S_r)er_i^t \leq 0. \]

\( er_i^t, j=1,2,\ldots,i \) is determined by (4) and (5) with \( er_i^{t+1} = er_i^{t+2} = \cdots = er_i^N = 0 \). The exact quitting time is closely related to the marginal cost of the exhaustible resource input and the production efficiency of each firm.

4. Further discussion

This section illustrates the relationship between the number of firms and the exhaustible resource input, including a discussion of a hierarchy of firms.

4.1. Symmetric cases

Under symmetric cases, \( \theta_1 = \theta_2 = \cdots = \theta_N \). Under equilibrium, the exhaustible resource input is identical for all firms. Alternatively, \( er_i^1 = er_i^2 = \cdots = er_i^N \) and \( \lambda_i^1 = \lambda_i^2 = \cdots = \lambda_i^N = \lambda_i \). According to (4) and (5), combining \( \theta_i = \theta_2 = \cdots = \theta_N \), we have
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\[ \frac{\partial([A - N\theta_i(e_t')^\alpha]_\theta_i(e_t')^\alpha - c(S_t)e_t')}{\partial e_t'} - \lambda_i = 0. \] (6)

\[ \lambda_i \geq 0 \] for all \( t \) is given by (5). Given the stock of exhaustible resource \( S_t \), by (5) and (6) we have the following conclusion:

**Proposition 3:** The exhaustible resource input \( e_t' \) and profit of each firm at any time monotonically decrease in \( N \), and \( \sum_{i=1}^{N} e_t' \) monotonically increases in \( N \).

**Remarks:** Proposition 3 means that the exhaustible resource inputs and firms’ profits monotonically decrease in \( N \), while the total exhaustible resource inputs \( \sum_{i=1}^{N} e_t' \) monotonically increases in \( N \). The exhaustible resource is exhausted sooner with more firms in this industry. This conclusion is highly consistent with the Courant model without exhaustible resources.

Under symmetric cases, the exhaustible resource inputs and profits are all captured. This conclusion provides theoretical supports for government decisions to protect exhaustible resources. In practice, many industries with exhaustible resource inputs are regulated to restrict the number of firms so that the exhaustible resource is available longer. This is an interesting application of the above conclusion.

### 4.2. Leader-follower position

This section analyzes firms’ strategies according to the hypothesis that \( \theta_1 > \theta_2 = \cdots = \theta_N \). In this case, the first firm has the highest production efficiency and assumes the leading position. The firms with lower production efficiency assume following positions. This sequence is denoted by \( \tau = \theta_1 - \theta_2 \); \( \theta_1 = \theta_2 = \cdots = \theta_N \) implies the relationship \( e_{r_1} = e_{r_2} = \cdots = e_{r_N} \) and \( \lambda_1^2 = \cdots = \lambda_N^2 = \lambda \) for all time. The first optimal conditions are restated as follows. For the first firm, we have:
\[ \alpha \theta_i (e_1^i)^{\alpha-1} [A - (N - 1) \theta_2 (e_2^i)^{\alpha} - 2 \theta_1 (e_1^i)^{\alpha}] - c(s_i) - \lambda_1^i = 0. \]  \tag{7}

For other firms, we have:

\[ \alpha \theta_i (e_1^i)^{\alpha-1} [A - (N - 1) \theta_i (e_1^i)^{\alpha} - \theta_1 (e_1^i)^{\alpha}] - c(s_i) - \lambda_i = 0. \]  \tag{8}

\( \lambda_i \geq 0 \) and \( \lambda_1^i \geq 0 \) for all \( t \) is given by (5), (7) and (8) directly imply the following conclusion.

**Proposition 4:** If the first firm has a production efficiency advantage, then the first firm’s exhaustible resource input increases while those of other firms decrease in the degree of production efficiency advantage.

**Proof:** (7) and (8) yield \( \frac{\partial e_1^i}{\partial \tau} > 0 \) and \( \frac{\partial e_1^i}{\partial \tau} < 0 \) for any time \( t \) and \( i = 2, 3, \ldots, N \). The conclusion is achieved, and the proof is complete. \[\Box\]

**Remarks:** This conclusion means that the degree of production efficiency advantage is positively related to the production gap between the more efficient and less efficient firms.

Under \( \theta_i = \theta_1 = \cdots = \theta_N \), Proposition 3 also holds. Given \( \theta_2 = \theta_i = \cdots = \theta_N \), (8) indicates the relationship \( \frac{\partial}{\partial \tau} \sum_{i=1}^{N} e_1^i < 0 \). Alternatively, the total level of exhaustible resource inputs is reduced when there is an advantage in production efficiency. This means that a firm with an advantage in production efficiency decreases the consuming exhaustible resources in this industry.

5. Concluding Remarks

This study addresses the effects of exhaustible resource inputs on production and finds that the exhaustible properties have deep effects on firms’ strategies, including target output.
quantity and quitting time. This article shows that that price of the end products increases as the exhaustible resource input decreases with time. The quitting time is discussed. The number of firms in the industry has major effects on the production schedule and the time at which firms quit the industry.

The theoretical conclusions and some hypotheses regarding the model are consistent with the empirical evidence of Boyce and Nostbakken (2011), such as the cost of exhaustible resources and production strategies of firms. The results achieved in this paper are also consistent with those in Van der Ploeg (2010).

Exhaustible resource inputs create many complicated economic phenomena. This study only addresses one output; when multiple outputs are considered, the situation becomes more complicated, which is a topic for further research.

6. References


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Appendix

Proof of Proposition 2

Because \( \frac{dc(S_i)}{dt} = -\sum_{i=1}^{N} e'_i \frac{dc(S_i)}{\partial S_i} \), (4) and (5) imply that \(-c(S_i) - \lambda'_i\) is decreased in \(t\). Model (4) implies that the term \( \alpha \theta(A - \sum_{i=1}^{N} \theta_i^{(x)} - \theta_i^{(y)})\) is increased in \(t\). On the other hand, (4) implies that \( \alpha \theta(A - \sum_{i=1}^{N} \theta_i^{(x)} - \theta_i^{(y)})>0 \). The concavity of the profit function indicates \( \frac{d}{de'_i} [\alpha \theta(A - \sum_{i=1}^{N} \theta_i^{(x)} - \theta_i^{(y)})] \) decreases with \( e'_i \). The formulations

\[ \frac{d}{de'_i} [\alpha \theta(A - \sum_{i=1}^{N} \theta_i^{(x)} - \theta_i^{(y)})] \]

imply that the exhaustible resource input \( e'_i \) monotonically decreases in \(t\) for all \(i=1,2,\cdots,N\).

Because the exhaustible resource input \( e'_i \) monotonically decreases in \(t\) for all \(i=1,2,\cdots,N\), \( p_i = A - \sum_{i=1}^{N} \theta_i^{(x)} \) shows that the price \( p_i \) monotonically increases in \(t\).

By the envelop theorem, because \(-c(S_i) - \lambda'_i\) is decreased in \(t\), we see that \( \pi'_i \) monotonically decreases in \(t\) for all \(i=1,2,\cdots,N\).

Conclusions are achieved, and the proof is complete. ■

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